

Inclusion-Exclusion Principle for Sharia Portfolio Optimization: Implications for Islamic Financial Governance

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ABSTRAK

Penelitian ini mengusulkan penerapan Prinsip Inklusi-Eksklusi (Inclusion-Exclusion Principle/PIE) dalam matematika diskrit sebagai kerangka deterministik untuk optimalisasi portofolio investasi syariah di Indonesia. Berbeda dengan pendekatan optimasi portofolio konvensional yang umumnya menggunakan model probabilistik, simulasi stokastik, atau fuzzy logic, metode yang diusulkan memanfaatkan operasi union sets untuk mengintegrasikan berbagai instrumen keuangan syariah sekaligus menghilangkan duplikasi dan tumpang tindih aset. Penelitian ini menggunakan pendekatan konseptual dan pemodelan matematis berbasis teori himpunan serta Prinsip Inklusi-Eksklusi untuk membangun portofolio optimal yang terdiri atas sukuk, mudharabah, musyarakah, murabahah, ijarah, dan aset wakaf produktif. Hasil penelitian menunjukkan bahwa model berbasis PIE mampu meningkatkan diversifikasi portofolio melalui perhitungan kardinalitas gabungan aset secara akurat sehingga setiap aset hanya dihitung satu kali dalam struktur portofolio. Pendekatan ini meningkatkan efisiensi alokasi, mengurangi redundansi, serta memperkuat kepatuhan syariah dengan meminimalkan potensi tumpang tindih yang dapat menimbulkan persoalan tata kelola. Selain itu, model yang dihasilkan menawarkan kerangka yang transparan dan mudah diaudit untuk mendukung pengawasan regulator dan tata kelola keuangan syariah.

Kata Kunci: Diversifikasi Deterministik; Optimalisasi Portofolio Syariah; Prinsip Inklusi-Eksklusi; Tata Kelola Keuangan Syariah; Union Sets

ABSTRACT

This study proposes the application of the Inclusion-Exclusion Principle (PIE) from discrete mathematics as a deterministic framework for optimizing Sharia-compliant investment portfolios in Indonesia. Unlike conventional portfolio optimization approaches that rely on probabilistic models, stochastic simulations, or fuzzy logic, the proposed method employs union set operations to integrate various Islamic financial instruments while eliminating asset duplication and overlapping exposures. The study adopts a conceptual and mathematical modeling approach based on set theory and the Inclusion-Exclusion Principle to construct an optimal portfolio consisting of sukuk, mudharabah, musyarakah, murabahah, ijarah, and productive waqf assets. The findings demonstrate that the PIE-based model improves portfolio diversification by accurately calculating the cardinality of combined asset classes and ensuring that each asset is counted only once within the portfolio structure. This approach enhances allocation efficiency, reduces redundancy, and strengthens Sharia compliance by minimizing the potential for hidden overlaps that may generate governance concerns. Furthermore, the model offers a transparent and auditable framework that supports regulatory oversight and Islamic financial governance. The study contributes to the growing literature on Islamic finance by introducing a mathematical governance perspective for portfolio optimization and provides practical implications for regulators and Islamic financial institutions seeking to improve portfolio management, risk monitoring, and Sharia compliance in an increasingly complex financial environment.

Keywords: Deterministic Diversification, Inclusion-Exclusion Principle, Islamic Financial Governance, Sharia Portfolio Optimization, Union Sets

INTRODUCTION

The rapid expansion of Islamic finance has significantly increased the diversity of Sharia-compliant financial instruments available to investors worldwide. Over the past two decades, Islamic financial markets have evolved beyond traditional banking activities to encompass sukuk, Islamic mutual funds, *mudharabah* and *musyarakah* financing, *ijarah*-based investments, and various forms of Islamic social finance. This development has enhanced financial inclusion and created new opportunities for portfolio diversification while ensuring compliance with Islamic principles (Omotubora, 2024; Tidjani & Madouri, 2024; Shashwat et al., 2025). In Indonesia, the growth of Islamic finance has been particularly significant, supported by the expansion of Islamic banking, the increasing issuance of sovereign and corporate sukuk, and the development of productive waqf and other Sharia-based financial instruments. As the Islamic financial ecosystem becomes more complex, the challenge of constructing efficient and diversified Sharia-compliant portfolios has become increasingly important for investors, financial institutions, and regulators (Muslichah et al., 2023; McGregor & Alghamdi, 2024).

Portfolio diversification is a fundamental objective of investment management because it helps reduce concentration risk and improve financial resilience. Conventional portfolio optimization models generally rely on probabilistic frameworks such as Modern Portfolio Theory (MPT), stochastic simulations, risk-return optimization techniques, and more recently, fuzzy logic and machine-learning approaches. These methods have contributed substantially to portfolio management by assisting investors in allocating assets under conditions of uncertainty. However, their application in Islamic finance presents unique challenges. Unlike conventional portfolios, Islamic investment portfolios operate within a restricted investment universe governed by the prohibition of *riba* (interest), *gharar* (excessive uncertainty), and *maysir* (speculation). As a result, portfolio construction must simultaneously pursue diversification, risk management, and Sharia compliance (Tuzzahroh & Laela, 2022; Farezy et al., 2025; Maksum et al., 2025).

The increasing variety of Sharia-compliant assets does not automatically guarantee effective diversification. Islamic financial instruments often share similar economic functions, financing objectives, underlying assets, or sectoral exposures despite being structured under different contractual arrangements (Aminuddin et al., 2025; Judijanto et al., 2025; Wibowo et al., 2025). For example, sukuk, *mudharabah*, *musyarakah*, and *ijarah* instruments may exhibit overlapping characteristics within a portfolio. Such overlaps can create hidden redundancies that reduce diversification efficiency and potentially distort portfolio evaluation. From a governance perspective, overlapping exposures may also complicate risk monitoring, asset allocation decisions, and compliance assessments. Consequently, identifying and managing redundancy among Sharia-compliant assets has become an important issue in Islamic portfolio management.

Existing studies on Islamic portfolio optimization have predominantly focused on risk-return efficiency, uncertainty management, and investment decision-making using probabilistic, stochastic, and fuzzy-based approaches. While these methods provide valuable insights into portfolio performance, they primarily emphasize market uncertainty and future asset behavior. Comparatively less attention has been devoted to deterministic approaches that examine the structural composition of portfolio assets. In particular, the issue of overlapping asset exposures within Sharia-compliant portfolios remains underexplored in the literature. As Islamic financial markets continue to expand, the absence of a systematic framework for identifying and eliminating portfolio redundancy may limit the effectiveness of diversification strategies and weaken portfolio transparency.

To address this gap, this study proposes the application of the Inclusion–Exclusion Principle (PIE) from discrete mathematics as a deterministic framework for Sharia portfolio optimization. The Inclusion–Exclusion Principle provides a systematic method for calculating the cardinality of combined sets by accounting for overlapping elements and preventing double counting. Through the operation of *union sets*, multiple categories of Islamic financial assets can be integrated into a unified portfolio structure while ensuring that overlapping components are accurately identified and excluded from duplication (Fadillah & Lubis, 2024; Muljaningsih et al., 2024). Unlike probabilistic optimization models that rely on assumptions regarding future market behavior, the proposed approach focuses on the structural relationships among portfolio assets and offers a transparent mathematical mechanism for diversification.

The relevance of the Inclusion–Exclusion Principle extends beyond portfolio construction to broader issues of Islamic financial governance. Transparency, accountability, and effective risk management are central elements of governance within Islamic finance (Sairally, 2024; Shi et al., 2025). By providing a clear framework for identifying asset overlaps and measuring effective diversification, the proposed model can support more transparent portfolio evaluation and facilitate governance oversight. Furthermore, the deterministic nature of the approach is consistent with the Islamic emphasis on clarity, fairness, and responsible financial management, thereby contributing to stronger compliance with Sharia principles and the objectives of *Maqashid al-Shari'ah* (Azizah & Salam, 2021).

Therefore, this study aims to develop a theoretical framework for Sharia portfolio optimization based on the Inclusion–Exclusion Principle and *union set* operations. The study further examines the implications of this approach for diversification efficiency, portfolio transparency, and Islamic financial governance in Indonesia. By introducing a set-theoretical perspective into Islamic portfolio management, this research contributes to the growing literature on Islamic finance and offers an alternative analytical framework for regulators, portfolio managers, and Islamic financial institutions seeking to improve asset allocation strategies in increasingly complex Sharia-compliant financial markets.

METHOD

This study employs a conceptual and mathematical modeling approach to examine the application of the Inclusion–Exclusion Principle (PIE) in optimizing Sharia-compliant investment portfolios. The research is grounded in discrete mathematics, particularly set theory, and aims to develop a deterministic framework for portfolio diversification within the context of Islamic finance. Rather than relying on probabilistic assumptions or stochastic forecasting, the study focuses on the structural relationships among Islamic financial assets through the operation of *union sets* and the Inclusion–Exclusion Principle.

The study adopts a qualitative-descriptive design supported by mathematical formulation (Creswell & Poth, 2023). The conceptual analysis is developed through a review of literature related to portfolio diversification, Islamic financial governance, set theory, and Sharia-compliant investment instruments. Relevant sources include academic journal articles, textbooks on discrete mathematics and financial management, reports from Bank Indonesia and the Financial Services Authority (OJK), as well as regulatory frameworks and fatwas governing Islamic financial activities in Indonesia (Almusaed et al., 2025). These sources provide the theoretical foundation for integrating mathematical concepts with Islamic portfolio management practices.

The mathematical framework is constructed by representing Islamic financial instruments as sets containing unique asset characteristics. The portfolio is modeled as the union of multiple Sharia-compliant asset classes, including sukuk, *mudharabah*, *musyarakah*, *murabahah*, *ijarah*, and productive waqf assets. To avoid double counting and portfolio redundancy, the Inclusion–Exclusion Principle is applied to calculate the effective cardinality of the combined asset portfolio. For two sets, the model is expressed as:

$$|A \cup B| = |A| + |B| - |A \cap B|$$

For multiple asset classes, the generalized Inclusion–Exclusion Principle is employed to account for higher-order intersections among sets. This procedure enables the identification and elimination of overlapping exposures, thereby producing a more accurate measure of portfolio diversification.

The analytical process consists of four stages. First, Sharia-compliant financial instruments are classified into distinct asset sets based on their contractual and economic characteristics. Second, potential intersections among asset sets are identified to determine overlapping components. Third, the Inclusion–Exclusion Principle is applied to calculate the effective portfolio cardinality after eliminating redundancy. Finally, the resulting portfolio structure is evaluated from the perspective of diversification efficiency and Islamic financial governance, particularly regarding transparency, accountability, risk monitoring, and Sharia compliance. By combining conceptual analysis with mathematical modeling, this study provides a deterministic framework for portfolio optimization that complements existing probabilistic and fuzzy-based approaches. The method offers a transparent and auditable mechanism for integrating Islamic financial assets while supporting the governance objectives of diversification, efficiency, and compliance with Sharia principles.

RESULTS AND DISCUSSION

Conceptual Construction of a PIE-Based Sharia Portfolio

The proposed model applies the Inclusion–Exclusion Principle (PIE) as a deterministic framework for constructing diversified Sharia-compliant portfolios. In this approach, each Islamic financial instrument is represented as a set containing unique asset characteristics and investment exposures. Let S_1 denote sovereign sukuk, S_2 corporate sukuk, S_3 *mudharabah-musyarakah* financing, S_4 *murabahah* contracts, and S_5 productive waqf assets. The overall portfolio is represented as the union of these sets:

$$P = S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5$$

A fundamental challenge in portfolio construction is the existence of overlapping exposures among asset classes. Although Islamic financial instruments differ contractually, they may share common economic sectors, underlying assets, financing objectives, or risk characteristics. If these overlaps are ignored, portfolio diversification may be overstated because certain exposures are counted more than once.

To address this issue, the Inclusion–Exclusion Principle calculates the effective cardinality of the portfolio by systematically removing overlapping elements. For multiple sets, the generalized formula can be expressed as:

$$|P| = \sum |S_i| - \sum |S_i \cap S_j| + \sum |S_i \cap S_j \cap S_k| - \dots$$

This calculation produces a more accurate representation of portfolio composition by ensuring that each unique exposure is counted only once. Consequently, the resulting portfolio reflects actual diversification rather than apparent diversification generated by duplicated asset structures.

The application of PIE is particularly relevant in Islamic finance because portfolio transparency and asset traceability are closely associated with Sharia governance requirements. A portfolio that contains hidden overlaps may create inefficiencies in risk allocation and complicate compliance monitoring. By eliminating duplication, the proposed framework strengthens both diversification quality and governance transparency.

Portfolio Diversification through Union Set Operations

The analysis demonstrates that union set operations provide a systematic mechanism for maximizing diversification within Sharia-compliant portfolios. Traditional diversification strategies generally focus on increasing the number of assets held within a portfolio. However, a larger number of assets does not necessarily imply greater diversification if many of those assets contain overlapping exposures.

Through the Inclusion–Exclusion Principle, diversification is evaluated based on the number of unique exposures rather than the number of instruments alone. This distinction is particularly important in Islamic financial markets, where multiple instruments may be linked to similar underlying economic activities (Razak & Asutay, 2022; Holle et al., 2023; Rantaprasaja, 2025). For example, sovereign sukuk and corporate sukuk may both be associated with infrastructure sectors, while *murabahah* and *musyarakah* financing may target similar business activities. Without overlap adjustment, portfolio managers may incorrectly assume that diversification has increased when, in reality, exposure concentration remains high.

The proposed framework therefore shifts the focus of portfolio optimization from asset quantity to asset uniqueness. This approach is consistent with the objective of reducing concentration risk while maintaining compliance with Sharia principles. Furthermore, the model provides a transparent method for identifying redundancy, enabling portfolio managers to construct more efficient allocation strategies.

From a mathematical perspective, the model enhances diversification efficiency because every additional asset class contributes only its unique component to the overall portfolio. As a result, portfolio expansion becomes more disciplined and less susceptible to redundancy. This characteristic is particularly valuable in emerging Islamic financial markets where product innovation often leads to the introduction of instruments with partially overlapping economic functions.

Governance Implications of Deterministic Portfolio Optimization

Beyond diversification efficiency, the proposed model offers important implications for Islamic financial governance. Governance in Islamic finance emphasizes transparency, accountability, risk monitoring, and compliance with Sharia principles (Widiastuti et al., 2022; Napitupulu et al., 2024). These objectives require investment structures that are not only financially efficient but also clearly understandable and auditable.

One of the main advantages of the Inclusion–Exclusion Principle is its deterministic nature. Unlike stochastic optimization models, which depend on probability distributions and future market assumptions, the PIE framework operates through explicit mathematical relationships among asset sets. This characteristic improves transparency because portfolio

construction can be verified directly through observable asset classifications and overlap calculations.

The model also strengthens accountability by providing a clear explanation of how diversification is measured and how overlapping exposures are treated. Regulators, auditors, and Sharia supervisory boards can more easily evaluate portfolio structures because the methodology relies on objective mathematical procedures rather than subjective parameter estimation. Consequently, the framework supports stronger governance practices within Islamic financial institutions.

Another governance benefit relates to compliance monitoring. Overlapping exposures may obscure the true composition of portfolios and create difficulties in assessing Sharia compliance. By identifying and removing redundancy, the PIE approach enhances portfolio clarity and facilitates more effective supervision by regulatory authorities and Sharia boards.

Comparative Analysis with Probabilistic and Fuzzy Approaches

Existing studies on Islamic portfolio optimization have largely employed probabilistic models, stochastic simulations, and fuzzy logic frameworks. These approaches are useful for addressing uncertainty and forecasting future market conditions. However, they generally focus on risk-return estimation rather than structural portfolio composition.

The PIE framework differs fundamentally because it addresses diversification as a set-theoretical problem rather than a probabilistic one. Instead of estimating future asset performance, the model evaluates the uniqueness of portfolio components and removes duplication. As a result, the framework complements rather than replaces conventional optimization methods.

Compared with fuzzy logic models, which rely on membership functions and subjective parameter selection, the Inclusion–Exclusion Principle provides a fully deterministic calculation process. This improves reproducibility and reduces methodological ambiguity. Similarly, compared with Monte Carlo simulations, the proposed model requires fewer assumptions regarding future market behavior, making it easier to implement and audit.

Practical Implications for Islamic Financial Governance

The findings suggest that the Inclusion–Exclusion Principle has practical relevance for regulators and Islamic financial institutions in Indonesia. The framework can be incorporated into portfolio monitoring systems to identify overlapping exposures among Sharia-compliant assets and improve diversification assessment. For financial regulators such as OJK and Bank Indonesia, the model offers a transparent analytical tool that can support macroprudential supervision of Islamic financial markets. By integrating PIE-based calculations into asset monitoring dashboards, regulators may obtain a clearer picture of portfolio concentration and interconnected exposures across the Islamic financial sector.

For Islamic financial institutions, the framework can enhance portfolio management by providing a systematic method for reducing redundancy and improving allocation efficiency. Moreover, the transparency of the model aligns closely with the governance objectives of accountability, auditability, and Sharia compliance. Overall, the proposed approach demonstrates that the Inclusion–Exclusion Principle can serve not only as a mathematical tool for portfolio optimization but also as a governance mechanism that strengthens transparency, diversification efficiency, and regulatory oversight within the Islamic financial system.

CONCLUSION

This study proposes the Inclusion–Exclusion Principle (PIE) as a deterministic mathematical framework for optimizing Sharia-compliant portfolios through the application of *union set* operations. The findings demonstrate that PIE provides a systematic mechanism for integrating various Islamic financial instruments while eliminating overlapping exposures and asset redundancies. By calculating the effective cardinality of combined asset classes, the model enables portfolio diversification based on unique exposures rather than merely increasing the number of instruments. Consequently, the proposed approach offers a more accurate representation of diversification within Islamic portfolios and contributes to more efficient asset allocation.

The study further reveals that the application of PIE extends beyond portfolio optimization and has important implications for Islamic financial governance. The deterministic nature of the model enhances transparency, accountability, and auditability by providing a clear mathematical procedure for identifying and removing overlapping portfolio components. This characteristic aligns closely with the governance objectives of Islamic finance, particularly in promoting Sharia compliance, risk monitoring, and responsible financial management. Compared with probabilistic, stochastic, and fuzzy-based approaches, the PIE framework offers a complementary perspective that emphasizes structural portfolio composition and diversification efficiency.

From a practical standpoint, the proposed framework may support Islamic financial institutions, portfolio managers, and regulatory authorities in improving portfolio monitoring and governance practices. The integration of PIE-based calculations into portfolio management systems can facilitate the detection of overlapping exposures, strengthen diversification assessment, and improve the transparency of investment decisions. These advantages are particularly relevant in Indonesia's rapidly expanding Islamic financial sector, where the increasing variety of Sharia-compliant instruments requires more sophisticated approaches to portfolio construction and oversight.

Despite its contributions, this study remains conceptual and relies on a theoretical modeling approach. The framework has not yet been tested using large-scale empirical portfolio data, and its effectiveness may vary depending on the characteristics of specific Islamic financial markets. Future research is therefore encouraged to conduct empirical validation using real-world data from Islamic banks, sukuk markets, mutual funds, and other Sharia-compliant investment vehicles. Further studies may also integrate the Inclusion–Exclusion Principle with risk-adjusted portfolio models, network analysis, artificial intelligence, or other discrete mathematical operations to develop more comprehensive frameworks for Islamic portfolio optimization and financial governance. Such efforts would contribute to strengthening both the theoretical foundations and practical applications of mathematical approaches within Islamic finance.

REFERENCES

- Almusaed, A., Almssad, A., & Yitmen, I. (2025). Qualitative Data Collection and Management. *Practice of Research Methodology in Civil Engineering and Architecture*, 417–454. https://doi.org/10.1007/978-3-031-97393-2_13

- Aminuddin, Wibowo, W., Choiri, A., Ariga, A., & Satibi, I. (2025). Kebijakan Sertifikasi Wakaf di Indonesia dalam Pembangunan Berkelanjutan: Peluang dan Tantangan. *PESHUM: Jurnal Pendidikan, Sosial Dan Humaniora*, 4(2), 2665–2676. <https://doi.org/10.56799/PESHUM.V4I2.7914>
- Azizah, S. N., & Salam, A. N. (2021). Determinants of Mother's Financial Literacy on Family Economic Welfare: Maqashid Sharia Analysis. *Journal Research of Social Science, Economics, and Management*, 1(2). <https://doi.org/10.59141/jrssem.v1i2.12>
- Creswell, J. W., & Poth, C. N. (2023). *Qualitative Inquiry and Research Design: Choosing Among Five Approaches*, 5th Edition. 552.
- Fadillah, A. N., & Lubis, D. (2024). The Influence of Islamic Financial Literacy, Islamic Financial Inclusion, and Financial Behavior on the Investment Decisions of Generation Z in West Java. *Falah: Jurnal Ekonomi Syariah*, 9(1), 01–16. <https://doi.org/10.22219/JES.V9I1.29286>
- Farezy, I., Ariani, Z., Hidayanti, N. F., Agustina, A., & Hulaimi, A. (2025). *Integration of Blockchain Technology in Islamic Finance: Opportunities and Challenges Towards Transparency and Sharia Compliance*. <https://journal.ummat.ac.id/index.php/iicece/article/view/31325>
- Holle, M. H., Asiyah, B. N., Zumaroh, Z., Koerniawati, D., Mohamad, S. I. S., & Pellu, A. (2023). The Relevance of Mosque Financial Inclusion and Economic Sustainability. *El-Qist: Journal of Islamic Economics and Business (JIEB)*, 13(2), 178–189. <https://doi.org/10.15642/elqist.2023.13.2.178-189>
- Judijanto, L., Sarif, A., & Ariyanti, R. (2025). Productive Waqf as a Sustainable Financial Instrument in the Perspective of Islamic SDGs. *West Science Islamic Studies*, 3(02), 106–110. <https://doi.org/10.58812/WSISS.V3I02.1833>
- Maksum, M., Muflih, M., Rohman, M. M., Injaz, F., & Sanrego, Y. D. (2025). Why Do Women Micro-Entrepreneurs Adopt Islamic Digital Financing? Perspectives from UTAUT, Trust and Sharia Compliance. *Journal of Islamic Marketing*, 16(12), 3509–3539. <https://doi.org/10.1108/JIMA-03-2023-0087>
- McGregor, S. L. T., & Alghamdi, A. K. H. (2024). Measuring Islamic Financial Literacy. *Financial Planning Research Journal*, 10(1). <https://doi.org/10.2478/FPRJ-2024-0003>
- Muljaningsih, S., Ekawaty, M., Anggraini, I. K., & Galuh, A. K. (2024). Empowering Personal Financial Management Through Islamic Spirituality and Financial Independence (Freedom). *Journal of International Conference Proceedings*, 7(5), 991–1006. <https://doi.org/10.32535/JICP.V7I5.3783>
- Muslichah, Putra, Y. H. S., Abdullah, R., & Usry, A. K. (2023). Islamic Financial Literacy and Financial Inclusion: Examining the Intervening Role of Financial Self-Efficacy. *International Journal of Islamic Economics and Finance (IJIEF)*, 6(2), 181–200. <https://doi.org/10.18196/IJIEF.V6I2.16384>
- Napitupulu, R. M., Sukmana, R., & Rusydiana, A. S. (2024). Governance of Islamic Social Finance: Learnings from Existing Literature. *International Journal of Islamic and Middle Eastern Finance and Management*, 17(3), 552–571. <https://doi.org/10.1108/IMEFM-06-2023-0222>
- Omotubora, A. (2024). AI For Women's Financial Inclusion—Analysis of Product Design and Policy Approaches in Nigeria. *Data & Policy*, 6, e66. <https://doi.org/10.1017/dap.2024.62>
- Rantaprasaja, L. (2025). Economic Empowerment Through Islamic Financial Inclusion: The Strategic Role of Universities in Synergy with Islamic Financial Institutions. *Diversity: Jurnal*

- Razzak, A. A., & Asutay, M. (2022). Financial Inclusion and Economic well-being: Evidence from Islamic Pawnbroking (Ar-Rahn) in Malaysia. *Research in International Business and Finance*, 59, 101557. <https://doi.org/10.1016/J.RIBAF.2021.101557>
- Sairally, B. S. (2024). Waqf: A Key Islamic Social Finance Instrument. *ISRA International Journal of Islamic Finance*, 16(S1), 1–3. <https://doi.org/10.55188/IJIF.V16IS1.778>
- Shashwat, S., Chhabra, M., & Giri, A. K. (2025). Environmental Externalities of Digital Financial Inclusion: a Green Growth Perspective. *Discover Sustainability*, 6(1), 1–14. <https://doi.org/10.1007/S43621-025-00862-1/TABLES/8>
- Shi, J., Firmansyah, E. A., Wang, Y., & Xu, W. (2025). Technological Innovation and Regulatory Harmonization in Islamic Finance: A Systematic Review and Machine Learning Analysis (2000–2023). *Journal of Islamic Accounting and Business Research*. <https://doi.org/10.1108/JIABR-01-2025-0026>
- Tidjani, C., & Madouri, A. (2024). Fintech, Financial Inclusion, and Sustainable Development in the African Region. *Frontiers in Applied Mathematics and Statistics*, 10, 1276218. <https://doi.org/10.3389/FAMS.2024.1276218/FULL>
- Tuzzahroh, F., & Laela, S. F. (2022). *Sharia Audit and Shariah Compliance of Islamic Financial Institutions: A Bibliometric Analysis*. Jurnal Ekonomi Syariah Teori Dan Terapan. https://www.researchgate.net/publication/366150022_Sharia_Audit_and_Shariah_Compliance_of_Islamic_Financial_Institutions_A_Bibliometric_Analysis
- Wibowo, W., Choiri, A., Ariga, A., & Aminuddin. (2025). Manajemen Wakaf Produktif dalam Mengatasi Masalah Kemiskinan di Indonesia. *J-CEKI: Jurnal Cendekia Ilmiah*, 4(3), 274–292. <https://doi.org/10.56799/JCEKI.V4I3.7755>
- Widiastuti, T., Ningsih, S., Prasetyo, A., Mawardi, I., Herianingrum, S., Robani, A., Al Mustofa, M. U., & Hady, A. F. (2022). Developing an Integrated Model of Islamic Social Finance: Toward an Effective Governance Framework. *Heliyon*, 8(9), e10383. <https://doi.org/10.1016/j.heliyon.2022.e10383>